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Solving Quadratic Programming Problems with The Criss – Cross Approach

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ABSTRACT:

A criss - cross approach is proposed in this study to solve the Fuzzy Linear Complementarity Problem (FLCP). It is also suggested how to solve the Fuzzy Quadratic Programming Problem (FQP). Trapezoidal fuzzy numbers represent the cost coefficients, constraint coefficients, and right-side coefficients in FQP. The KKT requirements are used to convert the Fuzzy Quadratic Programming Problem into a Fuzzy Linear Complementarity Problem (FLCP), and the criss - cross technique is suggested as a solution for the described model. Through an example, the efficacy of the suggested approach is demonstrated.

1. INTRODUCTION

A general problem unifying bimatrix games, linear and quadratic programmes, is called the Linear Complementarity Problem (LCP). Numerous extensive advantages have resulted from the study of LCP. Given the n×n matrix M and the n-dimensional column vector q, the Linear Complementarity Problem (LCP) consists of finding non – negative unknown vectors w and z which satisfying,

$$w - Mz = q \tag{1.1}$$

$$w_i,\ z_i \geq 0, \text{ for } i = 1,2,\dots n \tag{1.2}$$

and $$w_i z_i = 0, \text{ for } i = 1,2,\dots n \tag{1.3}$$

Given the non negativity of the vectors w and z, (1.3) requires that $w_i z_i = 0$ for i = 1,2,…,n. Two such vectors are said to be complementarity. A solution (w,z) to the LCP is called a complementarity feasible solution. In linear complementarity problem there is no objective function is to be optimized.

One of the most extensively researched mathematical programming problems is the LCP. Numerous disciplines of operations research explore and use LCP and quadratic programming. A number of techniques were created to address linear complementarity issues. Those techniques make use of various pivot rules. Be aware that there are a number of non-pivot approaches as well. By expanding the ellipsoid

approach for this problem, Kozlov et al. [8] provided a polynomial method for QP. These days, polynomial time interior point algorithms for QP and LCP are presented in a number of works. Morris and Todd [9] formulated the QP problem and LCP of oriented matroids, providing a combinatorial generalisation of QP and LCP.

The Index technique under fuzzy environment was introduced in 2012 [11] as a means of solving LCPs; however, the article states that the index method is not applicable to LCPs where the diagonal pivot is zero. In this study, the criss - cross algorithm can solve any type of LCP. With a few changes, this method's procedure is the same as the index method. Thus, the modified index approach is another name for this technique. The resultant solution has to be supplementary. In 2022, Irene Hepzibah.R And Sudha. N. [6], discussed to solve Neutrosophic Fuzzy Quadratic Programming Problem using Taylor Series Approach. Xianfeng Ding et al. [13] introduced the new Method for Solving Full Fuzzy Quadratic Programming Problem in 2023.

Lemke presented a complementarity pivoting algorithm in 1968 [10] to address complementarity issues. In this research, we suggested a new approach, called the Linear Complementarity Approach, to solve the Fuzzy Quadratic Programming Problem (FQP). Trapezoidal fuzzy numbers are the representation of cost coefficients, constraint coefficients, and right-side coefficients in FQP. When an unclear circumstance occurs, we employ the fuzzy idea in this instance. The provided fuzzy quadratic programming problem is transformed into a fuzzy linear complementarity problem using the KKT condition. After that, the crisscross method is used to solve the FLCP.

The structure of the paper is as follows: A fundamental understanding of the trapezoidal fuzzy number and its arithmetic operation is given in Section 2. In Section 3, the Fuzzy Linear Complementarity Problem is explained. An algorithm for addressing a fuzzy linear complementarity problem using a criss - cross approach is given in Section 4. An example demonstrates the efficacy of the suggested strategy in section 5. Section 6 presents a suggested approach for converting the FQPP into FLCP and yields the answer for the given FQPP. We finally wrap up the paper.

2. PRELIMINARIES

2.1 Fuzzy Set

A fuzzy set $\widetilde{D}$ is defined by $\widetilde{D} = \{(x, \mu_{\widetilde{D}}(x)) : x \in D, \mu_{\widetilde{D}}(x) \in [0,1]\}$. In the pair $(x, \mu_{\widetilde{D}}(x))$, the first element x belong to the classical set D, the second element $\mu_{\widetilde{D}}(x)$ belong to the closed interval [0, 1] called Membership function.

2.2 Fuzzy Number

The notion of fuzzy numbers was introduced by Dubois D. and Prade H. A fuzzy subset $\widetilde{D}$ of the real line R with membership function $\mu_{\widetilde{D}} : R \to [0,1]$ is called a fuzzy number if

i) A fuzzy set $\widetilde{D}$ is normal.

ii) $\widetilde{D}$ is fuzzy convex,
(i.e.) $\mu_{\widetilde{D}}[\eta x_1 + (1-\eta)x_2] \geq \mu_{\widetilde{D}}(x_1) \wedge \mu_{\widetilde{D}}(x_2), x_1, x_2 \in R, \forall \eta \in [0,1]$.

iii) $\mu_{\widetilde{D}}$ is upper continuous, and

iv) Supp $\widetilde{D}$ is bounded, where supp $\widetilde{D} = \{x \in R : \mu_{\widetilde{D}}(x) > 0\}$.

2.3 Trapezoidal Fuzzy Number

A Trapezoidal fuzzy number $\widetilde{D}$ is denoted as, $\widetilde{D} = (d_1, d_2, d_3, d_4)$ and is defined by the membership function as,

$$\mu_{\widetilde{D}}(x) = \begin{cases} \dfrac{(x-d_1)}{(d_2-d_1)} & for\ d_1 \leq x \leq d_2 \\ 1 & for\ d_2 \leq x \leq d_3 \\ \dfrac{(d_4-x)}{(d_4-d_3)} & for\ d_3 \leq x \leq d_4 \\ 0, & otherwise \end{cases}$$

2.4 The Fuzzy Arithmetic Operations on Trapezoidal Fuzzy Number under Function Principle

Let $\tilde{C} = (c_1, c_2, c_3, c_4)$ and $\tilde{D} = (d_1, d_2, d_3, d_4)$ are two trapezoidal fuzzy numbers with the condition that $c_1 \leq c_2 \leq c_3 \leq c_4$ and $d_1 \leq d_2 \leq d_3 \leq d_4$. If $c_1 = c_2 = c_3 = c_4$ and $d_1 = d_2 = d_3 = d_4$ then $\tilde{C}$ & $\tilde{D}$ are crisp numbers.

Then the fuzzy arithmetic operations under function principle is given by,

Addition:

$\tilde{C} + \tilde{D} = (c_1 + d_1, c_2 + d_2, c_3 + d_3, c_4 + d_4)$, where $c_1, c_2, c_3, c_4, d_1, d_2, d_3$ and d_4 are all any real numbers.

Subtraction:

$\tilde{C} - \tilde{D} = (c_1 - d_4, c_2 - d_3, c_3 - d_2, c_4 - d_1)$, where $c_1, c_2, c_3, c_4, d_1, d_2, d_3$ and d_4 are all any real numbers.

Multiplication:

$$\tilde{C} \cdot \tilde{D} = \begin{matrix} (\min(c_1d_1, c_1d_4, c_4d_1, c_4d_4),\ \min(c_2d_2, c_2d_3, c_3d_2, c_3d_3), \\ \max(c_2d_2, c_2d_3, c_3d_2, c_3d_3), \max(c_1d_1, c_1d_4, c_4d_1, c_4d_4)) \end{matrix}$$

If $c_1, c_2, c_3, c_4, d_1, d_2, d_3$ and d_4 are all nonzero positive real numbers, then

$\tilde{C} \cdot \tilde{D} = (c_1 \cdot d_1, c_2 \cdot d_2, c_3 \cdot d_3, c_4 \cdot d_4)$

Scalar Multiplication:

Let, $\lambda \in R$, then $\lambda\tilde{D} = (\lambda d_1, \lambda d_2, \lambda d_3, \lambda d_4);\ \lambda \geq 0$

$\beta\tilde{D} = (\beta d_4, \beta d_3, \beta d_2, \beta d_1);\ \beta < 0$

Division:

$$\frac{\tilde{C}}{\tilde{D}} = \begin{matrix} \min\left(\frac{c_1}{d_1}, \frac{c_1}{d_4}, \frac{c_4}{d_1}, \frac{c_4}{d_4}\right),\ \min\left(\frac{c_2}{d_2}, \frac{c_2}{d_3}, \frac{c_3}{d_2}, \frac{c_3}{d_3}\right), \\ \max\left(\frac{c_2}{d_2}, \frac{c_2}{d_3}, \frac{c_3}{d_2}, \frac{c_3}{d_3}\right), \max\left(\frac{c_1}{d_1}, \frac{c_1}{d_4}, \frac{c_4}{d_1}, \frac{c_4}{d_4}\right) \end{matrix}$$

If $c_1, c_2, c_3, c_4, d_1, d_2, d_3$ and d_4 are all nonzero positive real numbers, then

$$\frac{\tilde{C}}{\tilde{D}} = \left(\frac{c_1}{d_4}, \frac{c_2}{d_3}, \frac{c_3}{d_2}, \frac{c_4}{d_1}\right)$$

2.5 Graded Mean Integration Representation Method

The process of converting fuzzy values to crisp values is called defuzzification. For a while now, defuzzification techniques have been extensively researched and used with fuzzy systems. These approaches' main goal was to extract a typical value from a given collection based on a set of predetermined characters. A relationship between the set of all fuzzy sets and the set of all real numbers is provided by the defuzzification approach.

Let $\tilde{C} = (c_1, c_2, c_3, c_4)$ be a trapezoidal fuzzy number, then the defuzzified value $\tilde{A}$ using the graded mean integration representation method [3] is given by

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$$P(\tilde{C}) = \frac{c_1 + 2c_2 + 2c_3 + c_4}{6}$$

This method is also used to ranking the trapezoidal fuzzy numbers to choose which is minimum and maximum.

3. FUZZY LINEAR COMPLEMENTARITY PROBLEM (FLCP)

Given a real nxn square matrix M and a nx1 real vector q, then the linear complementarity problem denoted by LCP(q,M) is to find real nx1 vector w, z such that

$$w - Mz = q \quad (3.1)$$

$$w_j \geq 0, z_j \geq 0 \text{ , for } j = 1,2,3,\ldots\ldots\ldots n \quad (3.2)$$

$$w_j z_j = 0 \text{ , for } j = 1,2,3,\ldots\ldots\ldots n \quad (3.3)$$

Here the pair (w_j, z_j) is said to be a pair of complementary variables.

A solution (w, z) to the above system is called a complementary feasible solution, if (w, z) is a basic feasible solution to (3.1) and (3.2) with one of the pair (w_j, z_j) is basic for j = 1, 2, 3,…n.

If q ≥0, then we immediately see that w = q, z = 0 is a solution to the linear complementarity problem. Assume that all parameters in (3.1) - (3.3) are fuzzy and are described by trapezoidal fuzzy numbers. Then, the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with trapezoidal fuzzy numbers.

$$\widetilde{w} - \widetilde{M}\tilde{z} = \tilde{q}$$

$$\widetilde{w}_j \geq 0, \tilde{z}_j \geq 0, j = 1,2,\ldots,n$$

$$\widetilde{w}_j\, \tilde{z}_j = 0, j = 1,2,\ldots,n$$

The pair $(\widetilde{w}_j\, \tilde{z}_j)$ is said to be a pair of fuzzy complementarity variables.

4. Criss - Cross Algorithm:

Step 1:

Initialization:

Input $(\tilde{q}^0, \widetilde{M}^0) = (\tilde{q}, \widetilde{M})$. Set v = 0.

Step 2:

Test for Termination:

If $\tilde{q}^v \geq 0$, then stop. $\tilde{z}^v = 0$ solves $(\tilde{q}^v, \widetilde{M}^v)$. That is $(\widetilde{w}, \tilde{z}) = (\tilde{q}, 0)$.

Step 3:

Choose Pivot Row:

(i) Leaving Variable Selection:
Choose the index ψ, so that, $\psi = \min\{i/\tilde{q}_i^v < 0\}$. If there is no ψ, then Stop. A feasible complementarity solution found.

(ii) Entering Variable selection:
Diagonal Pivot:
If, $m_{\psi\psi} < 0$, then make a diagonal pivot and repeat the procedure.
If, $m_{\psi\psi} = 0$, select an exchange pivot.
Exchange Pivot:
If, $m_{\psi\psi} = 0$, then, $\gamma = \min\,(j\colon m_{\psi j} < 0 \text{ or } m_{j\psi} > 0)$
If there is no γ, then Stop. LCP is infeasible.
If there is an γ, then make an exchange pivot on (γ, ψ) & (ψ, γ) and repeat the procedure.

5. Numerical Example:

Consider the Fuzzy Linear Complementarity Problems$(\tilde{q}, \widetilde{M})$, with triangular fuzzy number is,

$$\widetilde{M} = \begin{pmatrix} [0,0,0,0] & [-1.3,-1.1,-0.9,-0.7] & [1.7,1.9,2.1,2.3] \\ [1.7,1.9,2.1,2.3] & [0,0,0,0] & [-2.3,-2.1,-1.9,-1.7] \\ [-1.3,-1.1,-0.9,-0.7] & [0.7,0.9,1.1,1.3] & [0,0,0,0] \end{pmatrix},$$

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$$\tilde{q} = \begin{pmatrix} [-3.3,-3.1,-2.9,-2.7] \\ [5.7,5.9,6.1,6.3] \\ [-1.3,-1.1,-0.9,-0.7] \end{pmatrix}$$

The above problem can be written in the simplex table format

Basic Variables	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{q}$
$\tilde{w}_1$	[0.7,0.9,1.1,1.3]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[0.7,0.9,1.1,1.3]	[-2.3,-2.1,-1.9,-1.7]	[-3.3,-3.1,-2.9,-2.7]
$\tilde{w}_2$	[0,0,0,0]	[0.7,0.9,1.1,1.3]	[0,0,0,0]	[-2.3,-2.1,-1.9,-1.7]	[0,0,0,0]	[1.7,1.9,2.1,2.3]	[5.7,5.9,6.1,6.3]
$\tilde{w}_3$	[0,0,0,0]	[0,0,0,0]	[0.7,0.9,1.1,1.3]	**[0.7,0.9,1.1,1.3]**	[-1.3,-1.1,-0.9,-0.7]	[0,0,0,0]	[-1.3,-1.1,-0.9,-0.7]
$\tilde{w}_1$	[0.7,0.9,1.1,1.3]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[0.7,0.9,1.1,1.3]	**[-2.3,-2.1,-1.9,-1.7]**	[-3.3,-3.1,-2.9,-2.7]
$\tilde{w}_2$	[0,0,0,0]	[0.7,0.9,1.1,1.3]	[0.85,1.5,2.5,4.4]	[-1.5,-0.6,0.6,2.7]	[-4.4,-2.5,-1.5,-0.85]	[1.7,1.9,2.1,2.3]	[1.3,3.4,4.6,5.5]
$\tilde{z}_1$	[0,0,0,0]	[0,0,0,0]	[0.5,0.8,1.2,1.9]	[0.5,0.8,1.2,1.9]	[-1.9,-1.2,-0.8,-0.5]	[0,0,0,0]	[-1.9,-1.2,-0.8,-0.5]
$\tilde{z}_3$	[-0.8,-0.6,-0.4,-0.3]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[-0.8,-0.6,-0.4,-0.3]	[0.7,0.9,1.1,1.4]	[1.2,1.4,1.6,1.9]
$\tilde{w}_2$	[0.5,0.8,1.3,1.8]	[0.7,0.9,1.1,1.3]	[0.85,1.5,2.5,4.4]	[-1.5,-0.6,0.6,2.7]	[-3.9,-1.7,-0.2,0.95]	[-1.5,-0.4,0.4,1.1]	[-3.1,0,1.9,3.5]
$\tilde{z}_1$	[0,0,0,0]	[0,0,0,0]	[0.5,0.8,1.2,1.9]	[0.5,0.8,1.2,1.9]	**[-1.9,-1.2,-0.8,-0.5]**	[0,0,0,0]	[-1.9,-1.2,-0.8,-0.5]
$\tilde{z}_3$	[-0.8,-0.6,-0.4,-0.3]	[0,0,0,0]	[-3,-0.9,-0.3,-0.1]	[-3,-0.9,-0.3,-0.1]	[-0.7,-0.3,0.5,2.7]	[0.7,0.9,1.1,1.4]	[1.3,1.7,2.5,4.9]
$\tilde{w}_2$	**[0.5,0.8,1.3,1.8]**	[0.7,0.9,1.1,1.3]	[-14,-1.1,2.4,8]	[-16.3,-3.2,0.5,6.3]	[-4.5,-1.6,2.4,15.8]	[-1.5,-0.4,0.4,1.1]	[-6.7,0.1,4.5,18.3]
$\tilde{z}_2$	[0,0,0,0]	[0,0,0,0]	[-3.8,-1.5,-0.67,-0.3]	[-3.8,-1.5,-0.67,-0.3]	[0.3,0.67,1.5,3.8]	[0,0,0,0]	[0.3,0.67,1.5,3.8]
$\tilde{z}_3$	[-0.7,-0.36,0.56,2.6]	[0.1,0.3,0.8,2.1]	[-25.4,-1.7,1.5,12.7]	[-29.1,-3.3,0.1,10]	[-7.9,-1.5,2.3,28]	[-3.9,-0.7,0.7,2.9]	[-9.4,1.73,5.9,34.2]
$\tilde{w}_1$	[0.3,0.6,1.6,3.6]	[0.4,0.7,1.4,2.6]	[-28,-1.4,3,16]	[-32.6,-4,0.6,12.6]	[-9,-2,3,31.6]	[-3,-0.5,0.5,2.2]	[-13.4,0.08,5.6,36.6]
$\tilde{z}_2$	[0,0,0,0]	[0,0,0,0]	[-3.8,-1.5,-0.67,-0.3]	[-3.8,-1.5,-0.67,-0.3]	[0.3,0.67,1.5,3.8]	[0,0,0,0]	[0.3,0.67,1.5,3.8]

Then the solution of the given Fuzzy Linear Complementarity Problem is given by,

$\tilde{w}_1 = [-13.4,0.08,5.6,36.6], \tilde{w}_2 = [0,0,0,0], \tilde{w}_3 = [0,0,0,0];\ \tilde{z}_1 = [0,0,0,0], \tilde{z}_2 = [0.3,0.67,1.5,3.8], \tilde{z}_3 = [-9.4,1.73,5.9,34.2]$

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6. Procedure for convert the Fuzzy Quadratic Programming Problem (FQPP) into Fuzzy Linear Complementarity Problem (FLCP)

Let us consider the following FQPP,

$$Minimize\ \tilde{f}(\tilde{X}) = \tilde{C}\tilde{X} + \frac{1}{2}\tilde{X}^T\tilde{H}\tilde{X}$$

Subject to the constraints

$$\tilde{A}\tilde{X} \leq \tilde{b}$$
$$\tilde{X} \geq 0.$$

where $\tilde{C}$ is an n – vector of fuzzy numbers, $\tilde{b}$ is an m – vector, $\tilde{A}$ is an mxn fuzzy matrix and $\tilde{H}$ is an nxn fuzzy symmetric matrix. Let $\tilde{Y}$ denote the vector of slack variables and $\tilde{u}, \tilde{v}$ be the Lagrangian multiplier vectors of the constraints $\tilde{A}\tilde{X} \leq \tilde{b}\ and\ \tilde{X} \geq 0$, respectively. The Kuhn – Tucker conditions can then be written as,

$$\tilde{A}\tilde{X} + \tilde{Y} = \tilde{b}$$
$$-\tilde{H}\tilde{X} - \tilde{A}^T\tilde{u} + \tilde{v} = \tilde{C}$$
$$\tilde{X}^T\tilde{v} = 0, \tilde{u}^T\tilde{Y} = 0$$
$$\tilde{X}, \tilde{Y}, \tilde{u}, \tilde{v} \geq 0.$$

Now,

Here

$$\tilde{M} = \begin{bmatrix} \tilde{0} & -\tilde{A} \\ \tilde{A}^T & \tilde{H} \end{bmatrix}, \tilde{q} = \begin{bmatrix} \tilde{b} \\ \tilde{c} \end{bmatrix}, \tilde{w} = \begin{bmatrix} \tilde{Y} \\ \tilde{v} \end{bmatrix}\ and\ \tilde{Z} = \begin{bmatrix} \tilde{u} \\ \tilde{X} \end{bmatrix}$$

The Kuhn – Tucker conditions can be expressed as the LCP,

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q}$$
$$\tilde{W}^T\tilde{Z} = 0$$
$$(\tilde{W}, \tilde{Z}) \geq 0.$$

Thus the given FQPP is converted into the above FLCP.

Illustrative Example

Consider the following Fuzzy Quadratic Programming Problem (FQPP)

$$\text{Minimize } \tilde{z} = -\tilde{3}\tilde{x}_1 + \tilde{x}_1^2 + \tilde{2}\tilde{x}_1\tilde{x}_2 + \tilde{3}\tilde{x}_2^2$$

Subject to the constraints,

$$\tilde{2}\tilde{x}_1 + \tilde{x}_2 \leq \tilde{6}$$

$$\tilde{x}_1 - \tilde{4}\tilde{x}_2 \leq \tilde{0}$$

$$\tilde{x}_1, \tilde{x}_2 \geq \tilde{0}.$$

$$\tilde{A} = \begin{bmatrix} \tilde{2} & \tilde{1} \\ \tilde{1} & -\tilde{4} \end{bmatrix}, \ \tilde{H} = \begin{bmatrix} \tilde{2} & \tilde{2} \\ \tilde{2} & \tilde{6} \end{bmatrix}, \ \tilde{b} = \begin{bmatrix} \tilde{6} \\ \tilde{0} \end{bmatrix}, \tilde{c} = \begin{bmatrix} -\tilde{3} \\ \tilde{0} \end{bmatrix}$$

$$\tilde{M} = \begin{bmatrix} 0 & 0 & -\tilde{2} & -\tilde{1} \\ 0 & 0 & -\tilde{1} & \tilde{4} \\ \tilde{2} & \tilde{1} & \tilde{2} & \tilde{2} \\ \tilde{1} & -\tilde{4} & \tilde{2} & \tilde{6} \end{bmatrix}, \tilde{q} = \begin{bmatrix} \tilde{6} \\ \tilde{0} \\ -\tilde{3} \\ \tilde{0} \end{bmatrix}$$

The trapezoidal fuzzy number is expressed as,

$$\tilde{0} = [0,0,0,0], \tilde{1} = [0.7,0.9,1.1,1.3], \tilde{2} = [1.7,1.9,2.1,2.3], \tilde{3} = [2.7,2.9,3.1,3.3]$$

$$\tilde{4} = [3.7,3.9,4.1,4.3], \tilde{6} = [5.7,5.9,6.1,6.3]$$

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The above problem can be written in the simplex table format.

Basic Variables	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{w}_4$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{z}_4$	$\tilde{q}$
$\tilde{w}_1$	[0.7,0.9,1.1,1.3]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[1.7,1.9,2.1,2.3]	[0.7,0.9,2.1,2.3]	[5.7,5.9,6.1,6.3]
$\tilde{w}_2$	[0,0,0,0]	[0.7,0.9,2.1,2.3]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[0.7,0.9,1.1,1.3]	[-4.3,-4.1,-3.9,-3.7]	[0,0,0,0]
$\tilde{w}_3$	[0,0,0,0]	[0,0,0,0]	[0.7,0.9,1.1,1.3]	[0,0,0,0]	[-2.3,-2.1,-1.9,-1.7]	[-1.3,-1.1,-0.9,-0.7]	**[-2.3,-2.1,-1.9,-1.7]**	[-2.3,-2.1,-1.9,-1.7]	[-3.3,-3.1,-2.9,-2.7]
$\tilde{w}_4$	[0,0,0,0]	[0,0,0,0]	[0,0,0,0]	[0.7,0.9,1.1,1.3]	[-1.3,-1.1,-0.9,-0.7]	[3.7,3.9,4.1,4.3]	[-2.3,-2.1,-1.9,-1.7]	[-6.3,-6.1,-5.9,-5.7]	[0,0,0,0]
$\tilde{w}_1$	[0.7,0.9,1.1,1.3]	[0,0,0,0]	[0.5,0.8,1.3,1.8]	[0,0,0,0]	[-3.2,-2.3,-1.7,-1.2]	[-1.8,-1.3,-0.8,-0.5]	[-1.5,-0.4,0.4,1.1]	[-2.5,-1.4,-0.6,0.1]	[1.3,2.5,3.4,4.3]
$\tilde{w}_2$	[0,0,0,0]	[0.7,0.9,1.1,1.3]	[0.2,0.4,0.7,1]	[0,0,0,0]	[-1.8,-1.2,-0.8,-0.5]	**[-1,-0.7,-0.4,-0.2]**	[-1.1,-0.3,0.3,0.8]	[-6.1,-5.3,-4.7,-4.2]	[-2.5,-1.8,-1.3,-0.8]
$\tilde{z}_3$	[0,0,0,0]	[0,0,0,0]	[-0.8,-0.6,-0.4,-0.3]	[0,0,0,0]	[0.7,0.9,1.1,1.4]	[0.3,0.4,0.6,0.8]	[0.7,0.9,1.1,1.4]	[0.7,0.9,1.1,1.4]	[1.2,1.4,1.6,1.9]
$\tilde{w}_4$	[0,0,0,0]	[0,0,0,0]	[-1.8,-1.3,-0.8,-0.5]	[0.7,0.9,1.1,1.3]	[-0.1,0.6,1.4,2.5]	[4.2,4.7,5.4,6.1]	[-1.1,-0.4,0.4,1.5]	[-5.1,-4.4,-3.6,-2.5]	[2,2.7,3.4,4.4]
$\tilde{w}_1$	[0.7,0.9,1.1,1.3]	[-11.7,-3.6,-1,-0.4]	[-8.5,-1.5,0.8,1.7]	[0,0,0,0]	[-2.9,-1.4,2.2,15]	[-1.7,-0.8,1.5,8.5]	[-8.7,-1.4,1.4,11]	[-0.4,4,16.7,55]	[1.7,4,9.3,26.8]
$\tilde{z}_2$	[0,0,0,0]	[-6.5,-2.8,-1.3,-0.7]	[-5,-1.8,-0.6,-0.2]	[0,0,0,0]	[0.5,1.1,3,9]	[0.2,0.6,1.8,5]	[-4,-0.8,0.8,5.5]	[4.2,6.7,13.3,30.5]	[0.8,1.9,4.5,12.5]
$\tilde{z}_3$	[0,0,0,0]	[0.2,0.5,1.7,5.2]	[-0.7,-0.4,0.7,3.7]	[0,0,0,0]	[-6.5,-0.9,0.7,1.2]	[-3.7,-0.7,0.4,0.7]	[-3.7,0.4,1.6,4.6]	[-23.3,-7.1,-1.6,0.1]	[-8.8,-1.3,0.8,1.7]
$\tilde{w}_4$	[0,0,0,0]	[2.9,6.1,15.1,39.7]	[-1,1.5,8.9,30]	[0.7,0.9,1.1,1.3]	[-55,-15.6,-3.8,0.4]	[-26.3,-5,2.6,5.3]	[-34.7,-4.7,4.7,26]	**[-191,-76.2,-35,-20]**	[-74.3,-21.6,-5.5,1]

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$\tilde{w}_1$	[0.7,0.9,1.1,1.3][-12.5,-3.2,5.7,109][-11.2,-1.4,5.8,84][-0.03,0.04,0.5,3.8][-157,-8,2,16][-73,-2.5,2.7,25][-102,-3,3,82][-527,-33,15,59]	[-32,-1,9,75]
$\tilde{z}_2$	[0,0,0,0] [-6.4,-2.1,4,60] [-6.5,-1.7,3.4,46] [0,0.1,0.4,2.1] [-85,-4.2,2.7,10] [-39.5,-0.7,3,14] [-56,-2.1,2.1,46] [-289,-23,10,30]	[-16,-2,4,42]
$\tilde{z}_3$	[0,0,0,0] [-46.8,-2.3,1.5,5.4] [-36,-2.5,0.7,5] [-1.6,-0.2,-0.02,0] [-7,-0.8,3.5,67] [-11,-1,1.1,31] [-34,-0.3,2,44] [-24,-6,14,224]	[-11,-1.2,3,34]
$\tilde{z}_4$	[0,0,0,0][-2,-0.4,-0.1,-0.02][-1.5,-0.3,-0.02,0.05][-0.1,-0.03,-0.01,0][-0.02,0.05,0.4,3][-0.3,-0.1,0.1,1.3][-1.3,-0.1,0.1,1.7][0.1,0.5,2.2,9.6]	[-0.1,0.07,0.3,1.4]

Here the bold values are referred as the pivot element.

Hence the optimal solution of the given FQPP is $\tilde{X}_1$= [-11,-1.2,3,34], $\tilde{X}_2$= [-0.1,0.07,0.3,1.4] and Minimize $\tilde{z}$ = [-1403.6,-16.7,14.2,1402.3]

CONCLUSION

This research proposes a novel fuzzy linear complementarity problem-solving method for fuzzy quadratic programming problems. The method for transforming the Fuzzy Linear Complementarity Problem (FLCP) from the Fuzzy Quadratic Programming Problem (FQPP) is proposed. In this case, the proposed criss-cross technique uses a trapezoidal fuzzy number to solve the provided FLCP and FQPP. Any kind of fuzzy linear complementarity problem can be resolved with this criss-cross algorithm.

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