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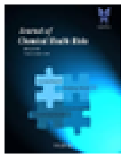

On Somewhat Neutrosophic Soft Regular Semi Continuous Functions

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Abstract

In this paper the concept of somewhat neutrosophic soft regular semi continuous functions, somewhat neutrosophic soft regular semi-open functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. More examples are given to illustrate the concepts introduced in this paper.

Key words : $SNSRSC$, $SNSRS$-O, $NSRS$-dense, $WSNSRSO$, $NSRS$-resolvable, $NSRS$-irresolvable.
AMS (2000) subject classification: 54A40, 03E72.

1 Introduction

The fuzzy set was described by Zadeh [21] and Chang [5] took on the task of defining its topological structure as fuzzy topological space. In 1983, Atanassow [1] initiated intuitionistic fuzzy set and coker [6] created intuitionistic fuzzy set in a topology. In 1999, Molodtsov [11] initiated the soft set theory and applied in different fields. Shabir and Naz [13] presented soft topological spaces. Smarandache [14, 15, 16] introduced the concept of neutrosophic set. Maji [10] has introduced a combined concept neutrosophic soft set (NSS). Later, this concept has been modified by Deli and Broumi [7]. The concept of regular semiopen set was introduced by Cameron [4] in 1978. Later, Vadivel and Elavarasan [8, 17] presented soft regular semiopen sets and soft regular semi continuous. In this concept has been generalized

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to neutrosophic soft setting. Recently, Vijayalakshmi et. al., [18, 19, 20] introduced the concept of neutrosophic soft regular semiopen (semiclosed) sets, neutrosophic soft regular semi continuous, neutrosophic soft regular semi irresolute, neutrosophic soft regular semi-$T_{1/2}$ space, neutrosophic soft regular semi homeomorphisms and neutrosophic soft weakly regular semi continuous in neutrosophic soft topological spaces. In this paper the concept of somewhat neutrosophic soft regular semi continuous functions, somewhat neutrosophic soft regular semi-open functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. More examples are given to illustrate the concepts introduced in this paper.

2 Preliminaries

In this section, we recollect some relevant basic preliminaries about Neutrosophic soft sets and its operations.

Definition 2.1. [14] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. That is $T_A; I_A; F_A : X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$ and so, $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

Definition 2.2. [11] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F : A \to P(U)$ is a mapping.

Definition 2.3. [10] Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all Ns of U. Then for $A \subseteq E$, a pair (F, A) is called an Nss over U, where $F : A \to NS(U)$ is a mapping.

This concept has been modified by Deli and Broumi [**?**, 7, 12] as given below.

Definition 2.4. [7, 12] Let X be an initial universe set and E be a set of parameters. Let $NS(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $(\tilde{F}, E)$ over X is a set defined by a set valued function $\tilde{F}$ representing a mapping $\tilde{F} : E \to NS(X)$ where $\tilde{F}$ is called approximate function of the neutrosophic soft set $(\tilde{F}, E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $NS(X)$ and therefore it can be written as a set of ordered pairs, $(\tilde{F}, E) = \{(e; \left\langle x; T_{\tilde{F}(e)}(x); I_{\tilde{F}(e)}(x); F_{\tilde{F}(e)}(x) \right\rangle : x \in U) : e \in E\}$ where

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$T_{\tilde{F}(e)}(x); I_{\tilde{F}(e)}(x); F_{\tilde{F}(e)}(x) \in [0;1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\tilde{F}(e)$. Since supremum of each $T; I; F$ is 1 so the inequality $0 \leq T_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2.5. [7, 12] Let $(\tilde{F}, E)$ be neutrosophic soft set over the universe set X. The complement of a neutrosophic soft set $(\tilde{F}, E)$ is denoted by $(\tilde{F}, E)^c$ and is defined by $(\tilde{F}, E)^c = \{(e; \{\left\langle x; F_{\tilde{F}(e)}(x); 1 - I_{\tilde{F}(e)}(x); T_{\tilde{F}(e)}(x) \right\rangle : x \in U\}) : e \in E\}$.

Definition 2.6. [7, 10, 12] Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two Nss's over the common universe X. Then $(\tilde{F}, E)$ is said to be the neutrosophic soft subset of $(\tilde{G}, E)$ if $\forall e \in E$ and $\forall x \in X$, $T_{\tilde{F}}(e)(x) \leq T_{\tilde{G}}(e)(x)$, $I_{\tilde{F}}(e)(x) \leq I_{\tilde{G}}(e)(x)$, $F_{\tilde{F}}(e)(x) \geq F_{\tilde{F}(e)}(x)$. We write $(\tilde{F}, E) \subseteq (\tilde{G}, E)$ and then $(\tilde{G}, E)$ is the neutrosophic soft superset of $(\tilde{F}, E)$.

Definition 2.7. [7, 10, 12] Let $(\tilde{F_1}, E)$ and $(\tilde{F_2}, E)$ be two Nss's over the common universe X. Then their union is denoted by $(\tilde{F_1}, E) \cup (\tilde{F_2}, E) = (\tilde{F_3}, E)$ and is defined as: $(\tilde{F_3}, E) = \{(e, \{\left\langle x, T_{\tilde{F_3}(e)}(x); I_{\tilde{F_3}(e)}(x); F_{\tilde{F_3}(e)}(x) \right\rangle : x \in X\}) : e \in E\}$ where
$T_{\tilde{F_3}(e)}(x) = max\{T_{\tilde{F_1}(e)}(x), T_{\tilde{F_2}(e)}(x)\}$,
$I_{\tilde{F_3}(e)}(x) = max\{I_{\tilde{F_1}(e)}(x), I_{\tilde{F_2}(e)}(x)\}$,
$F_{\tilde{F_3}(e)}(x) = min\{F_{\tilde{F_1}(e)}(x), F_{\tilde{F_2}(e)}(x)\}$.
Their intersection is denoted by $(\tilde{F_1}, E) \cap (\tilde{F_2}, E) = (\tilde{F_4}, E)$ and is defined as : $(\tilde{F_4}, E) = \{(e, \{\left\langle x, T_{\tilde{F_4}(e)}(x); I_{\tilde{F_4}(e)}(x); F_{\tilde{F_4}(e)}(x) \right\rangle : x \in X\}) : e \in E\}$ where
$T_{\tilde{F_4}(e)}(x) = min\{T_{\tilde{F_1}(e)}(x), T_{\tilde{F_2}(e)}(x)\}$,
$I_{\tilde{F_4}(e)}(x) = min\{I_{\tilde{F_1}(e)}(x), I_{\tilde{F_2}(e)}(x)\}$,
$F_{\tilde{F_4}(e)}(x) = max\{F_{\tilde{F_1}(e)}(x), F_{\tilde{F_2}(e)}(x)\}$.

Definition 2.8. [2, 12] A neutrosophic soft set $(\tilde{F}, E)$ over X is said to be null neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 0; I_{\tilde{F}(e)}(x) = 0; F_{\tilde{F}(e)}(x) = 1, x \in X, e \in E$. It is denoted by $0_{(X,E)}$. A neutrosophic soft set $(\tilde{F}, E)$ over X is said to be absolute neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 1; I_{\tilde{F}(e)}(x) = 1; F_{\tilde{F}(e)}(x) = 0, x \in X, e \in E$. It is denoted by $1_{(X,E)}$. Clearly, ${0_{(X,E)}}^c = 1_{(X,E)}$ and ${1_{(X,E)}}^c = 0_{(X,E)}$.

Definition 2.9. [2, 12] Let $Nss(X, E)$ be the family of all neutrosophic soft sets over X via parameters in E and $\tau \subset NSS(X, E)$. Then τ is called neutrosophic soft topology on (X, E) if the following conditions are satisfied.
(i) $0_{(X,E)}$, $1_{(X,E)} \in \tau$.
(ii) the intersection of any finite number of members of τ also belongs to τ.

(iii) the union of any collection of members of τ belongs to τ.
Then the triplet (X,τ,E) is called a neutrosophic soft topological space (for short, nsts). Every member of τ is called neutrosophic soft open set (**for short, nsos**). An Nss is called Neutrosophic soft closed (**for short, nscs**) iff its complement is nsos.

Definition 2.10. [2, 12] Let (X,τ,E) be a neutrosophic soft topological space over X and $(\tilde{F},E)\in NSS(X,E)$ be arbitrary. Then

(1) the neutrosophic soft interior of $(\tilde{F},E)$ is denoted by $NSInt((\tilde{F},E))$ and is defined as : $NSInt((\tilde{F},E))=\cup\{(\tilde{G},E):(\tilde{G},E)\text{is neutrosophic soft open and } (\tilde{G},E)\subset(\tilde{F},E)\}$.
i.e., it is the union of all neutrosophic soft open subsets of $(\tilde{F},E)$.

(2) the neutrosophic soft closure of $(\tilde{F},E)$ is denoted by $NSCl((\tilde{F},E))$ and is derined as follows $NSCl((\tilde{F},E))=\cap\{(\tilde{G},E):(\tilde{G},E)\text{is neutrosophic soft closed and } (\tilde{F},E)\subset(\tilde{G},E)\}$.
i.e., it is the union of all neutrosophic soft closed supersets of $(\tilde{F},E)$).

Definition 2.11. A nsts (X,τ,E), and a Nss $(\tilde{R},E)$ over (U,E). Then $(\tilde{R},E)$ is called:

(i) neutrosophic soft regular open set (**for short, nsros**) [9] iff $(\tilde{R},E)=NSInt(NSCl(\tilde{R},E))$.

(ii) neutrosophic soft regular semi open set (**for short, nsrsos**) [18] if there exists a nsros $(\tilde{S},E)$ such that $(\tilde{S},E)\subseteq(\tilde{R},E)\subseteq NSCl((\tilde{S},E))$.

(iii) neutrosophic soft regular semi closed set (**for short, nsrscs**) [18] if there exists a nsrcs $(\tilde{S},E)$ such that $NSInt((\tilde{S},E))\subseteq(\tilde{R},E)\subseteq(\tilde{S},A)$

The complement of nsros, and nsrsos sets are called **nsrcs, and nsrscs** sets.

Definition 2.12. [18] Let (X,τ,E) be a nsts over (X,E) and $(\tilde{R},E)\in NSS(X,E)$ be arbitrary. Then the

(i) neutrosophic soft regular semi interior of $(\tilde{R},E)$ is denoted by **NSRSInt**$((\tilde{\mathbf{R}},\mathbf{E}))$ and is defined as : $NSRSInt((\tilde{R},E))=\cup\{(\tilde{S},E):(\tilde{S},E)\text{is nsrsos and}(\tilde{S},E)\subseteq(\tilde{R},E)\}$. i.e., it is the union of all nsrsos of $(\tilde{S},E)$.

(ii) neutrosophic soft regular semi closure of $(\tilde{R},E)$ is denoted by **NSRSCl**$((\tilde{\mathbf{R}},\mathbf{E}))$ and is defined as : $NSRSCl((\tilde{R},E))=\cap\{(\tilde{S},E):(\tilde{S},E)\text{is nsrscs and}(\tilde{S},E)\supseteq(\tilde{R},E)\}$. i.e., it is the union of all nsrscs of $(\tilde{R},E)$.

Definition 2.13. [3] Let (U, E, τ_U) and (V, E, τ_V) be two neutrosophic soft topological spaces. Then $(\varphi, \psi) : (U, E, \tau_U) \to (V, E, \tau_V)$ is said to be

(1) a neutrosophic soft continuous mapping if for each $(N, E) \in \tau_V$, the inverse image $(\varphi, \psi)^{-1}((N, E)) \in \tau_U$. i.e., the inverse image of each open Nss in (V, E, τ_V) is also open in (U, E, τ_U).

(2) a neutrosophic soft open mapping if for each $(M, E) \in \tau_U$, the image $(\varphi, \psi)((M, E)) \in \tau_V$.

(3) a neutrosophic soft closed mapping if for each $(M, E) \in \tau_U$, the image $(\varphi, \psi)((M, E)) \in \tau_V$.

Definition 2.14. [19] Let (X, τ, E) and (Y, σ, E) be two nsts's. A Neutrosophic soft function $f : X \to Y$ is said to be

(1) neutrosophic soft regular continuous (for short, NSRC) if for each nsos $(\tilde{R}, E)$ of Y, the inverse image $f^{-1}((\tilde{R}, E))$ is a nsros of X.

(2) neutrosophic soft regular semi continuous (for short, NSRSC) if for each nsos $(\tilde{R}, E)$ of Y, the inverse image $f^{-1}((\tilde{R}, E))$ is a nsrsos of X.

(3) neutrosophic soft regular semi irresolute (for short, NSRSI) if for each nsrso set $(\tilde{R}, E)$ of Y, the inverse image $f^{-1}((\tilde{R}, E))$ is a nsrsos of X.

(4) neutrosophic soft regular semi open function (for short, NSRS-O) if for each nsos $(\tilde{S}, E)$ of X, the image $f((\tilde{S}, E))$ is a nsrsos of Y.

(5) neutrosophic soft regular semi closed function (for short, NSRS-C) if for each nscs set $(\tilde{S}, E)$ of X, the image $f((\tilde{S}, E))$ is a nsrscs of Y.

3 Somewhat neutrosophic soft regular semi continuous functions

Definition 3.1. Let $(X,\ \tau, E)$ and $(Y,\ \sigma, E)$ be any two nsts's. A function $f : (X,\ \tau, E) \to (Y,\ \sigma, E)$ is called

(i) somewhat neutrosophic soft regular continuous (for short, $SNSRC$) if for each nsos $(\tilde{R}, E)$ of Y and $f^{-1}((\tilde{R}, E)) \neq 0_{(X,E)}$ there exists a nsro $(\tilde{S}, E) \neq 0_{(X,E)}$ of X, such that $(\tilde{S}, E) \subseteq f^{-1}((\tilde{R}, E))$.

(ii) somewhat neutrosophic soft regular semi continuous (for short, $SNSRSC$) if for each nsos $(\tilde{R}, E)$ of Y and $f^{-1}((\tilde{R}, E)) \neq 0_{(X,E)}$ there exists a nsrso $(\tilde{S}, E) \neq 0_{(X,E)}$ of X, such that $(\tilde{S}, E) \subseteq f^{-1}((\tilde{R}, E))$.

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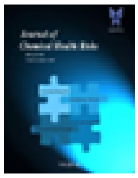

(iii) somewhat neutrosophic soft semi continuous (for short, $SNSSC$) if for each nsos $(\tilde{R}, E)$ of Y and $f^{-1}((\tilde{R}, E)) \neq 0_{(X,E)}$ there exists a nsso $(\tilde{S}, E) \neq 0_{(X,E)}$ of X, such that $(\tilde{S}, E) \subseteq f^{-1}((\tilde{R}, E))$.

Example 3.1
Let $X = \{a, b\}$, $E = \{e\}$, $\tau = \{0_{(X,E)}, 1_{(X,E)}, (\tilde{A}, E), (\tilde{B}, E)\}$, $Y = \{p, q\}$ and $\sigma = \{0_{(X,E)}, 1_{(X,E)}, (\tilde{D}, E)\}$, where $(\tilde{A}, E)$ and $(\tilde{B}, E)$ are Nss of X and $(\tilde{C}, E)$ is Nss of Y, defined as follows:

$(\tilde{A}, E) = \{e, \langle (a, 0.40, 0.50, 0.60)(b, 0.50, 0.50, 0.50) \rangle\}$,
$(\tilde{B}, E) = \{e, \langle (a, 0.40, 0.50, 0.40)(b, 0.50, 0.50, 0.50) \rangle\}$,
$(\tilde{C}, E) = \{e, \langle (a, 0.50, 0.50, 0.60)(b, 0.50, 0.50, 0.50) \rangle\}$, .
$(\tilde{D}, E) = \{e, \langle (a, 0.50, 0.50, 0.60)(b, 0.50, 0.50, 0.60) \rangle\}$, .

Clearly (X, τ, E) and (Y, σ, E) are nsts on X and Y. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then f is $SNSRSC$. Since a nsos $(\tilde{D}, E)$ of Y, $f^{-1}((\tilde{D}, E)) \neq 0_{(X,E)}$, there exist a nsrso set $(\tilde{C}, E) \neq 0_{(X,E)}$ such that $(\tilde{C}, E) \subseteq f^{-1}((\tilde{D}, E))$.

Remark 3.1.

(1) Every $SNSRC$ function is $SNSRSC$ but not conversely.

(2) Every $SNSRSC$ function is $SNSSC$ but not conversely.

Example 3.2 In Example 3.1, f is $SNSRSC$ but not $SNSRC$, because $(\tilde{C}, E)$ is not nsrc set in X such that $(\tilde{C}, E) \subseteq f^{-1}((\tilde{D}, E))$.

Example 3.3 Let $X = \{a, b\}$ and $\tau = \{0_{(X,E)}, 1_{(X,E)}, (\tilde{A}, E), (\tilde{B}, E)\}$, $Y = \{p, q\}$ and $\sigma = \{0_{(X,E)}, 1_{(X,E)}, (\tilde{D}, E)\}$, where $(\tilde{A}, E)$ and $(\tilde{B}, E)$ are Nss of X and $(\tilde{D}, E)$ is Nss of Y, defined as follows:

$(\tilde{A}, E) = \{e, \langle (a, 0.30, 0.50, 0.60)(b, 0.50, 0.50, 0.50) \rangle\}$,
$(\tilde{B}, E) = \{e, \langle (a, 0.60, 0.50, 0.50)(b, 0.50, 0.50, 0.50) \rangle\}$,
$(\tilde{C}, E) = \{e, \langle (a, 0.40, 0.50, 0.60)(b, 0.50, 0.50, 0.50) \rangle\}$,
$(\tilde{D}, E) = \{e, \langle (a, 0.40, 0.50, 0.60)(b, 0.50, 0.50, 0.50) \rangle\}$,

Clearly (X, τ, E) and (Y, σ, E) are nsts on X and Y. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then f is $SNSSC$ but not $SNSRSC$, Since for each nsos $(\tilde{D}, E))$, $f^{-1}((\tilde{D}, E)) \neq 0_{(X,E)}$, there exist a nsso set $(\tilde{C}, E) \neq 0_{(X,E)}$ (since $\exists$ a nsos $(\tilde{B}, E)$ such that $(\tilde{B}, E) \subseteq (\tilde{C}, E) \subseteq NSCl((\tilde{B}, E))$) such that $(\tilde{C}, E) \subseteq f^{-1}((\tilde{D}, E))$. but $(\tilde{C}, E)$ is not nsrso.

Definition 3.2. A Nss $(\tilde{R}, E)$ in a nsts (X, τ, E) is called $NSRS$-dense if there exists no nsrsc set $(\tilde{S}, E)$ such that $(\tilde{R}, E) \subset (\tilde{S}, E) \subset 1_{(X,E)}$.

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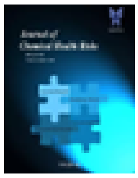

Proposition 3.1. Let $(X,\ \tau, E)$ and $(Y,\ \sigma, E)$ be any two nsts's and $f : (X,\ \tau, E) \to (Y,\ \sigma, E)$ be a function. Then the following are equivalent:

(1) f is $SNSRSC$,

(2) If $(\tilde{R}, E)$ is a nscs of Y such that $f^{-1}((\tilde{R}, E)) \neq 1_{(X,E)}$, then there exists a nsrsc set $(\tilde{S}, E) \neq 1_{(X,E)}$ of X such that $(\tilde{S}, E) \supseteq f^{-1}((\tilde{R}, E))$,

(3) If $(\tilde{R}, E)$ is a $NSRS$-dense set in X, then $f((\tilde{R}, E))$ is a $NSRS$-dense set in Y.

Proof. (1) $\Rightarrow$ (2). Suppose f is $SNSRSC$-continuous and $(\tilde{R}, E)$ is a nscs in Y such that $f^{-1}((\tilde{R}, E)) \neq 1_{(X,E)}$. Clearly $(\tilde{R}, E)^c$ is nsos and $f^{-1}((\tilde{R}, E)^c) = (f^{-1}((\tilde{R}, E)))^c \neq 0_{(X,E)}$ (since $f^{-1}((\tilde{R}, E)) \neq 1_{(X,E)}$). By (1), there exists a nsrso set $(\tilde{T}, E)$ in X such that $(\tilde{T}, E) \subseteq f^{-1}((\tilde{R}, E)^c) = (f^{-1}((\tilde{R}, E)))^c$. That is, $f^{-1}((\tilde{R}, E)) \subseteq (\tilde{T}, E)^c$. Clearly $(\tilde{T}, E)^c$ is nsrsc and taking $(\tilde{T}, E)^c = (\tilde{S}, E)$, we find that (1) $\Rightarrow$ (2) is proved.

(2) $\Rightarrow$ (3). Let $(\tilde{R}, E)$ be a $NSRS$-dense set in X and suppose $f((\tilde{R}, E))$ is not $NSRS$-dense in Y. Then there exists a nsrsc set $(\tilde{T}, E)$ (say) in Y such that

$$f((\tilde{R}, E)) \subset (\tilde{T}, E) \subset 1_{(X,E)} \tag{3.1}$$

Since $(\tilde{T}, E) \subset 1_{(X,E)}$, $f^{-1}((\tilde{T}, E)) \neq 1_{(X,E)}$ and so by (2) there exists a nsrsc set $(\tilde{U}, E)$ ($(\tilde{U}, E) \neq 1_{(X,E)}$) such that $(\tilde{U}, E) \supseteq f^{-1}((\tilde{T}, E)) \supseteq f^{-1}f((\tilde{R}, E)) \supseteq (\tilde{R}, E)$ [from 3.1]. That is, there exists a nsrsc set $(\tilde{U}, E)$ such that $(\tilde{U}, E) \supset (\tilde{R}, E)$ which is a contradiction to the assumption on $(\tilde{R}, E)$. Therefore (2) $\Rightarrow$ (3) is proved.

(3) $\Rightarrow$ (1). Suppose $(\tilde{R}, E) \neq 0_{(X,E)}$ be a nsro set and obviously nsos in Y and $f^{-1}((\tilde{R}, E)) \neq 0_{(X,E)}$. Suppose there exists no nsrso set $(\tilde{S}, E)$ in X such that $(\tilde{S}, E) \subseteq f^{-1}((\tilde{R}, E))$. Then $(f^{-1}((\tilde{R}, E)))^c$ is a Nss in X such that there is no nsrsc set $(\tilde{U}, E)$ in X with $(f^{-1}((\tilde{R}, E)))^c \subset (\tilde{U}, E) \subset 1_{(X,E)}$. [Otherwise $(f^{-1}((\tilde{R}, E)))^c \subset (\tilde{U}, E)$ implies $(\tilde{U}, E)^c \subset f^{-1}((\tilde{R}, E))$ and $(\tilde{U}, E)^c$ is nsrso, a contradiction]. That is, $(f^{-1}((\tilde{R}, E)))^c$ is a $NSRS$-dense set in X. Then by (3) $f((f^{-1}((\tilde{R}, E)))^c)$ is $NSRS$-dense in Y. But $f((f^{-1}((\tilde{R}, E)))^c) = f((f^{-1}((\tilde{R}, E)^c))) \subseteq (\tilde{R}, E)^c \subset 1_{(X,E)}$ and $f((f^{-1}((\tilde{R}, E)))^c) \subseteq (\tilde{R}, E)^c \subset 1_{(X,E)}$ implies $NSRSCl(f((f^{-1}((\tilde{R}, E)))^c)) \subseteq NSRSCl((\tilde{R}, E)^c)$. Then since $f([f^{-1}((\tilde{R}, E))]^c)$ is $NSRS$-dense, we have $1_{(X,E)} \subseteq NSRSCl((\tilde{R}, E)^c) = (\tilde{R}, E)^c$ [$(\tilde{R}, E)^c$ is nsrc set $\Rightarrow (\tilde{R}, E)^c$ is nsrsc]. This implies $(\tilde{R}, E) \subseteq 0_{(X,E)}$. That is, $(\tilde{R}, E) = 0_{(X,E)}$. But this a contradiction to the fact that $(\tilde{R}, E) \neq 0_{(X,E)}$. Therefore there exists a nsrso set $(\tilde{S}, E)$ in X such that $(\tilde{S}, E) \subseteq f^{-1}((\tilde{R}, E))$. This proves that f is $SNSRSC$. ■

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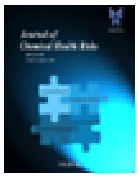

Remark 3.2. Product of nsrso sets is nsrso sets.

Proposition 3.2. Let $(X_1,\ \tau_1, E)$, $(X_2,\ \tau_2, E)$, $(Y_1,\ \sigma_1, E)$ and $(Y_2,\ \sigma_2, E)$ be nsts's such that X_1 is product related to X_2 and Y_1 is product related to Y_2. Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be $SNSRSC$. Then $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is a $SNSRSC$.

Proof. Let $(\tilde{R}, E) = \bigcup\limits_{i,\ j}((\tilde{R}_i, E) \times (\tilde{S}_j, E))$ be a nsos in $Y_1 \times Y_2$ where $(\tilde{R}_i, E)$ and $(\tilde{S}_j, E)$ are nsrso sets in Y_1 and Y_2 respectively. We can assume that $(\tilde{R}_i, E) \neq 0_{(X,E)}$ and $(\tilde{S}_j, E) \neq 0_{(X,E)}$. If any one is zero, that factor can be omitted. Now $(f_1 \times f_2)^{-1}((\tilde{R}, E)) = (f_1 \times f_2)^{-1}\big(\bigcup\limits_{i,j}((\tilde{R}_i, E) \times (\tilde{(S_j)}, E)\big) = \bigcup\limits_{i,j}\big(f_1 \times f_2\big)^{-1}((\tilde{R}_i, E) \times (\tilde{S}_j, E)) = \bigcup\limits_{i,j}\big(f_1^{-1}((\tilde{R}_i, E)) \times f_2^{-1}((\tilde{S}_j, E))\big)$. Since $f_1 : X_1 \to Y_1$ is $SNSRSC$ and $(\tilde{R}_i, E)$ is nsos in Y_1 and $f_1^{-1}((\tilde{R}_i, E)) \neq 0_{(X,E)}$, there exists a nsrso set $(\tilde{U}_i, E)$ in X_1 such that $(\tilde{U}_i, E) \subseteq f_1^{-1}((\tilde{R}_i, E))$. Since $f_2 : X_2 \to Y_2$ is $SNSRSC$ and $(\tilde{S}_j, E)$ is nsos in Y_2 and $f_2^{-1}((\tilde{S}_j, E)) \neq 0_{(X,E)}$, there exists a nsrso set $(\tilde{T}_j, E)$ in X_2 such that $(\tilde{T}_j, E) \subseteq f_2^{-1}((\tilde{S}_j, E))$. Therefore $(\tilde{U}_i, E) \times (\tilde{T}_j, E) \subseteq f_1^{-1}((\tilde{R}_i, E)) \times f_2^{-1}((\tilde{S}_j, E))$ and $(\tilde{U}_i, E) \times (\tilde{T}_j, E)$ is a nsrso set. Also $\bigcup\limits_{i,j}(\tilde{U}_i, E) \times (\tilde{T}_j, E) \subseteq \bigcup\limits_{i,j} f_1^{-1}((\tilde{R}_i, E)) \times f_2^{-1}((\tilde{S}_j, E))$ and $\bigcup\limits_{i,j}(\tilde{U}_i, E) \times (\tilde{T}_j, E)$ is nsrso in $X_1 \times X_2$. That is $\bigcup\limits_{i,j}(\tilde{U}_i, E) \times (\tilde{T}_j, E) \subseteq \bigcup\limits_{i,j}(f_1 \times f_1)^{-1}((\tilde{R}_i, E) \times (\tilde{S}_j, E))$. Thus $f_1 \times f_2$ is a $SNSRSC$. ■

Lemma 3.1. Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. If $(\tilde{R}, E)$ is a Nss of X and $(\tilde{S}, E)$ is a Nss of Y, then $g^{-1}((\tilde{R}, E) \times (\tilde{S}, E)) = (\tilde{R}, E) \cap f^{-1}((\tilde{S}, E))$.

Proof. Obvious. ■

Proposition 3.3. Let $f : (X,\ \tau, E) \to (Y,\ \sigma, E)$ be a function. Then, if the graph $g : X \to X \times Y$ of f is $SNSRSC$, then f is also $SNSRSC$

Proof. Let $(\tilde{R}, E) \neq 0_{(X,E)}$ be a nsos in Y. Then, by Lemma 3.1., we have $f^{-1}((\tilde{R}, E)) = 1_{(X,E)} \cap f^{-1}((\tilde{R}, E)) = g^{-1}(1_{(X,E)} \times (\tilde{R}, E))$. Since g is $SNSRSC$ and $1_{(X,E)} \times (\tilde{R}, E)(\neq 0_{(X,E)})$ is a nsos in $X \times Y$, there exists a nsrso set $(\tilde{S}, E) \neq 0_{(X,E)}$ (say) of X such that $(\tilde{S}, E) \subseteq g^{-1}(1_{(X,E)} \times (\tilde{R}, E)) = f^{-1}((\tilde{R}, E))$. This proves that f is a $SNSRSC$ function. ■

Proposition 3.4. Let X, X_1 and X_2 be nsts's and $p_i : X_1 \times X_2 \to X_i$ $(i = 1,\ 2)$ be any NSC function. If $f : X \to X_1 \times X_2$ is $SNSRSC$, then $p_i \circ f$ is also a $SNSRSC$ function for $i = 1,\ 2$.

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JCHR (2026) 16(1), 2717-2734 | ISSN:2251-6727

Proof. For any nsos $(\tilde{R},E) \neq 0_{(X,E)}$ of X_i, we have $(p_i \circ f)^{-1}((\tilde{R},E)) = f^{-1}\big(p_i^{-1}((\tilde{R},E))\big)$. Now $p_i^{-1}((\tilde{R},E)) \neq 0_{(X,E)}$ (since $(\tilde{R},E) \neq 0_{(X,E)}$). Since p_i is neutrosophic soft continuous, $p_i^{-1}((\tilde{R},E))$ is nsos and since f is a $SNSRSC$ function, then there exists a nsrso set $(\tilde{S},E)$ of X such that $(\tilde{S},E) \subseteq f^{-1}\big(p_i^{-1}((\tilde{R},E))\big) = (p_i \circ f)^{-1}((\tilde{R},E))$. Therefore $p_i \circ f$ is a $SNSRSC$ function. ■

Theorem 3.1. Let $(X, \ \tau, E)$ and $(Y, \ \sigma, E)$ be any two nsts's. If the function $f : (X, \ \tau, E) \to (Y, \ \sigma, E)$ is $SNSRSC$ and onto and if $NSRSInt((\tilde{R},E)) = 0_{(X,E)}$ for any Nss $(\tilde{R},E) \neq 0_{(X,E)}$ in $(X, \ \tau, E)$, then $NSRSInt\big(f((\tilde{R},E))\big) = 0_{(X,E)}$ in $(Y, \ \sigma, E)$.

Proof. Let $(\tilde{R},E) \neq 0_{(X,E)}$ be a Nss in $(X, \ \tau, E)$ such that $NSRSInt((\tilde{R},E)) = 0_{(X,E)}$. Then $(NSRSInt((\tilde{R},E)))^c = 1_{(X,E)}$ implies that $NSRSCl((\tilde{R},E)^c) = 1_{(X,E)}$. Since f is $SNSRSC$ and $(\tilde{R},E)^C$ is $NSRS$-dense in $(X, \ \tau, E)$, $f((\tilde{R},E)^c)$ is $NSRS$-dense in Y [by Proposition 3.1.]. That is, $NSRSCl(f((\tilde{R},E)^c)) = 1_{(X,E)}$. Then $NSRSCl((f((\tilde{R},E)))^c) = 1_{(X,E)}$. [since f is onto]. Therefore we have $(NSRSInr(f((\tilde{R},E)))^c = 1_{(X,E)}$ which implies that $NSRSInt(f((\tilde{R},E))) = 0_{(X,E)}$. Hence the proposition. ■

Definition 3.3. A nsts $(X, \ \tau, E)$ is called a $NSRS$-D-space if every nsos $(\tilde{R},E) \neq 0_{(X,E)}$ of X is $NSRS$-dense in X.

Proposition 3.5. If $f : (X, \ \tau, E) \to (Y, \ \sigma, E)$ is a $SNSRSC$ function from X onto Y and X is a $NSRS$-D-space, then Y is a $NSRS$-D-space.

Proof. Let $(\tilde{R},E) \neq 0_{(X,E)}$ be a nsos in Y. We want to show that $(\tilde{R},E)$ is $NSRS$-dense in Y. Suppose not. Then there exists a nsrsc set $(\tilde{S},E)$ in Y such that $(\tilde{R},E) \subset (\tilde{S},E) \subset 1_{(X,E)}$ and $f^{-1}((\tilde{R},E)) \subset f^{-1}((\tilde{S},E)) \subset f^{-1}(1_{(X,E)}) = 1_{(X,E)}$. Since $(\tilde{R},E) \neq 0_{(X,E)}$, $f^{-1}((\tilde{R},E)) \neq 0_{(X,E)}$ and since f is $SNSRSC$, there exists a nsrso set $(\tilde{T},E) \neq 0_{(X,E)}$ in X such that $(\tilde{T},E) \subset f^{-1}((\tilde{R},E))$ and $(\tilde{T},E) \subset f^{-1}((\tilde{R},E)) \subset f^{-1}((\tilde{S},E)) \subset NSRSCl(f^{-1}((\tilde{S},E))) \subset 1_{(X,E)}$. That is, $(\tilde{T},E) \subset NSRSCl(f^{-1}((\tilde{S},E))) \subset 1_{(X,E)}$. This contradicts the fact that $(X, \ \tau, E)$ is a $NSRS$-D-space and it proves that Y is a $NSRS$-D-space. ■

4 Somewhat neutrosophic soft regular semi open functions

Definition 4.1. Let $(X, \ \tau, E)$ and $(Y, \ \sigma, E)$ be any two nsts's. A function $f : (X, \ \tau, E) \to (Y, \ \sigma, E)$ is called somewhat neutrosophic soft regular

Journal of Chemical Health Risks
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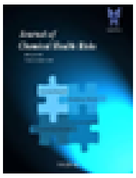

semi open function (for short, $SNSRS$-O) if for each nsos $(\tilde{R}, E)$ and $(\tilde{R}, E) \neq 0_{(X,E)}$, there exists a nsrso set $(\tilde{S}, E) \neq 0_{(X,E)}$ of $(Y, \ \sigma, E)$ such that $(\tilde{S}, E) \subseteq f((\tilde{R}, E))$.

Example 4.1 In Example 3.1, $f : Y \to X$ as $f(p) = a$ and $f(q) = b$, then f is $SNSRS$-O. Since a nsos $(\tilde{D}, E) \neq 0_{(X,E)}$ of Y, there exist a nsrso set $(\tilde{C}, E) \neq 0_{(X,E)}$ of X such that $(\tilde{C}, E) \subseteq f((\tilde{D}, E))$.

Proposition 4.1. Let $(X, \ \tau, E)$, $(Y, \ \sigma, E)$ and $(Z, \ \gamma, E)$ be nsts's. Suppose that $f : (X, \ \tau, E) \to (Y, \ \sigma, E)$ is neutrosophic soft open and $g : (Y, \ \sigma, E) \to (Z, \ \gamma, E)$ is $SNSRS$-O, then $g \circ f : (X, \ \tau, E) \to (Z, \ \gamma, E)$ is $SNSRS$-O.

Proof. Straightforward. ■

Theorem 4.1. Suppose that $(X, \ \tau, E)$ and $(Y, \ \sigma, E)$ be nsts's. Let $f : (X, \ \tau, E) \to (Y, \ \sigma, E)$ be an onto function. Then the following conditions are equivalent:

(i) f is $SNSRS$-O.

(ii) If $(\tilde{R}, E)$ is a $NSRS$-dense set in Y, then $f^{-1}((\tilde{R}, E))$ is a $NSRS$-dense set in X.

Proof. (1) $\Rightarrow$ (2). Assume (1). Suppose $(\tilde{R}, E)$ is a $NSRS$-dense set in Y. We want to show that $f^{-1}((\tilde{R}, E))$ is a $NSRS$-dense set in X. Suppose not. Then there exists a nsrsc set $(\tilde{S}, E)$ in X such that $f^{-1}((\tilde{R}, E)) \subset (\tilde{S}, E) \subset 1_{(X,E)}$. Since f is $SNSRS$-O and $(\tilde{S}, E)^c$ is nsrso, there exists a nsrso set $(\tilde{T}, E) \neq 0_{(X,E)}$ in Y such that $(\tilde{T}, E) \subset f((\tilde{S}, E)^c)$. Therefore $(\tilde{T}, E) \subset f((\tilde{S}, E)^c) \subset f(f^{-1}((\tilde{R}, E)^c)) \subseteq (\tilde{R}, E)^c$. That is, $(\tilde{R}, E) \subset (\tilde{T}, E)^c \subset 1_{(X,E)}$ (since $(\tilde{T}, E) \neq 0_{(X,E)}$). Now $(\tilde{T}, E)^c \neq 0_{(X,E)}$ is a nsrsc set and $(\tilde{R}, E) \subset (\tilde{T}, E)^c \subset 1_{(X,E)} \Rightarrow (\tilde{R}, E)$ is not a $NSRS$-dense set in Y, which is a contraction. Therefore $f^{-1}((\tilde{R}, E))$ must be a $NSRS$-dense set in X. This proves (1) $\Rightarrow$ (2).

(2) $\Rightarrow$ (1). Assume (2). Suppose for a nsro set (obviously nsos) $(\tilde{R}, E)$ and $(\tilde{R}, E) \neq 0_{(X,E)}$. We want to show that $NSRSInt(f((\tilde{R}, E))) \neq 0_{(X,E)}$. Suppose $NSRSInt(f((\tilde{R}, E))) = 0_{(X,E)}$. Now $NSRSC((f((\tilde{R}, E)))^c) = (NSRSInt(f((\tilde{R}, E))))^c = 1_{(X,E)}$. That is $(f((\tilde{R}, E)))^c$ is $NSRS$-dense in Y. Therefore by assumption (2), $f^{-1}((f((\tilde{R}, E)))^c)$ is $NSRS$-dense in X. But $f^{-1}((f((\tilde{R}, E)))^c) = (f^{-1}(f((\tilde{R}, E))))^c \subseteq (\tilde{R}, E)^c$. Then we have $NSRSCl(f^{-1}((f((\tilde{R}, E)))^c)) \subseteq NSRSCl((\tilde{R}, E)^c) = (\tilde{R}, E)^c$ [since $(\tilde{R}, E)^c$ is nsrc set $\Rightarrow (\tilde{R}, E)^c$ is nsrsc set]. That is, $1_{(X,E)} \subseteq (\tilde{R}, E)^c$. Then $(\tilde{R}, E) \subseteq 0_{(X,E)}$. That is, $(\tilde{R}, E) = 0_{(X,E)}$ which is a contradiction to $(\tilde{R}, E) \neq 0_{(X,E)}$. Therefore $NSRSInt(f((\tilde{R}, E))) \neq 0_{(X,E)}$. This proves that f is $SNSRS$-O. ■

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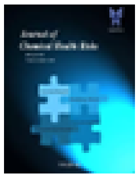

Theorem 4.2. Suppose (X, τ, E) and (Y, σ, E) be nsts's. Let the function $f : (X, \tau, E) \to (Y, \sigma, E)$ be a bijection function. Then the following are equivalent:

(1) f is $SNSRS\text{-}O$.

(2) If $(\tilde{R}, E)$ is a nscs in X such that $f((\tilde{R}, E)) \neq 1_{(X,E)}$, then there exists a nsrsc set $(\tilde{S}, E)$ in Y such that $(\tilde{S}, E) \neq 1_{(X,E)}$ and $(\tilde{S}, E) \supset f((\tilde{R}, E))$.

Proof. (1) $\Rightarrow$ (2): Let $(\tilde{S}, E)$ be a nscs on X such that $f((\tilde{S}, E)) \neq 1_{(X,E)}$. Since f is bijective and $(\tilde{S}, E)^c$ is a nsos on X, $f((\tilde{S}, E)^c) = (f((\tilde{S}, E)))^c \neq 0_{(X,E)}$. From the definition, there exists a nsrso set $(\tilde{U}, E) \neq 0_{(X,E)}$ on Y such that $(\tilde{U}, E) \subset f((\tilde{S}, E)^c) = (f((\tilde{S}, E)))^c$. Consequently, $f((\tilde{S}, E)) \subset (\tilde{U}, E)^c = (\tilde{V}, E) \neq 1_{(X,E)}$ and $(\tilde{V}, E)$ is a nsrsc set on Y.

(2) $\Rightarrow$ (1): Let $(\tilde{S}, E)$ be a nsos on X such that $f((\tilde{S}, E)) \neq 0_{(X,E)}$. Then $(\tilde{S}, E)^c$ is a nscs on X and $f((\tilde{S}, E)^c) \neq 1_{(X,E)}$. Hence there exists a nsrsc set $(\tilde{V}, E) \neq 1_{(X,E)}$ on Y such that $f((\tilde{S}, E)^c) \subset (\tilde{V}, E)$. Since f is bijective, $f((\tilde{S}, E)^c) = (f((\tilde{S}, E)))^c \subset (\tilde{V}, E)$. Thus $(\tilde{V}, E)^c \subset f((\tilde{S}, E))$ and $(\tilde{V}, E)^c \neq 0_{(X,E)}$ is a nsrso set on Y. Therefore, f is $SNSRS\text{-}O$. ■

Theorem 4.3. Let (X, τ, E) and (Y, σ, E) be any two nsts's. If the function $f : (X, \tau, E) \to (Y, \sigma, E)$ is $SNSRS$-open and if $NSRSInt((\tilde{R}, E)) = 0_{(X,E)}$ for any Nss $(\tilde{R}, E) \neq 0_{(X,E)}$ in (Y, σ, E), then $NSRSInt(f^{-1}((\tilde{R}, E))) = 0_{(X,E)}$ in (X, τ, E).

Proof. Let $(\tilde{R}, E) \neq 0_{(X,E)}$ be a Nss in (Y, σ, E) such that $NSRSInt((\tilde{R}, E)) = 0_{(X,E)}$. Then $(NSRSInt((\tilde{R}, E)))^c = 1_{(X,E)}$ implies that $NSRSCl((\tilde{R}, E)^c) = 1_{(X,E)}$. Since the function f is $SNSRS\text{-}O$ and $(\tilde{R}, E)^c$ is $NSRS$-dense in (Y, σ, E), $f^{-1}((\tilde{R}, E)^c)$ is $NSRS$-dense in (X, τ, E) [by Proposition 4.1.]. That is, $NSRSCl$ $(f^{-1}((\tilde{R}, E)^c)) = 1_{(X,E)}$. Then $NSRSCl([f^{-1}((\tilde{R}, E))]^c) = 1_{(X,E)}$.

Therefore $(NSRSInt(f^{-1}((\tilde{R}, E))))^c = 1_{(X,E)}$ implies that $NSRSInt(f^{-1}((\tilde{R}, E))) = 0_{(X,E)}$. ■

5 neutrosophic soft regular semi resolvable and irresolvable spaces

Definition 5.1. A nsts (X, τ, E) is said to be $NSRS$-resolvable if there exists a $NSRS$-dense set $(\tilde{R}, E) \neq 0_{(X,E)}$ in (X, τ) such that $NRSCl((\tilde{R}, E)^c) = 1_{(X,E)}$. Otherwise (X, τ, E) is called a $NSRS$-irresolvable space.

Theorem 5.1. A nsts (X, τ, E) is a $NSRS$-resolvable space if and only if (X, τ, E) has a pair of $NSRS$-dense sets $(\tilde{R}_1, E)$ and $(\tilde{R}_2, E)$ such that $(\tilde{R}_1, E) \subseteq (\tilde{R}_2, E)^c$.

Proof. Let $(X,\ \tau, E)$ be a $NSRS$-resolvable space. Suppose that for all $NSRS$-dense sets $(\tilde{R}_i, E)$ and $(\tilde{R}_j, E)$, we have $(\tilde{R}_i, E) \nsubseteq (\tilde{R}_j, E)^c$. Then we have $(\tilde{R}_i, E) \supset (\tilde{R}_j, E)^c$ for some i and j. Then, we have $NSRSCl((\tilde{R}_i, E)) \supset NSRSCl((\tilde{R}_j, E)^c)$ which implies that $1_{(X,E)} \supset NSRSCl((\tilde{R}_j, E)^c)$. Then $NSRSCl((\tilde{R}_j, E)^c) \neq 1_{(X,E)}$. Also $(\tilde{R}_j, E) \supset (\tilde{R}_i, E)^c$. Then $NSRSCl((\tilde{R}_j, E)) \supset NSRSCl((\tilde{R}_i, E)^c)$ which implies that $1_{(X,E)} \supset NSRSCl((\tilde{R}_i, E)^c)$. Then $NSRSCl((\tilde{R}_i, E)^c) \neq 1_{(X,E)}$. Hence $NSRSCl((\tilde{R}_i, E)) = 1_{(X,E)}$, but $NSRSCl((\tilde{R}_i, E)^c) \neq 1_{(X,E)}$ for all $NSRS$-dense sets $(\tilde{R}_i, E)$ in $(X,\ \tau, E)$, which is a contradiction to $(X,\ \tau, E)$ being a $NSRS$-resolvable space. Therefore $(X,\ \tau, E)$ has a pair of $NSRS$-dense sets $(\tilde{R}_1, E)$ and $(\tilde{R}_2, E)$ such that $(\tilde{R}_1, E) \subseteq (\tilde{R}_2, E)^c$.

Conversely, suppose that the nsts (X, τ, E) has a pair of $NSRS$-dense sets $(\tilde{R}_1, E)$ and $(\tilde{R}_2, E)$, such that $(\tilde{R}_1, E) \subseteq (\tilde{R}_2, E)^c$. We want to show that $(X,\ \tau, E)$ is $NSRS$-resolvable. Suppose that $(X,\ \tau, E)$ is a $NSRS$-irresolvable space. Then for all $NSRS$-dense sets $(\tilde{R}_i, E)$ in $(X,\ \tau, E)$, we have $NSRSCl((\tilde{R}_i, E)^c) \neq 1_{(X,E)}$. In particular $NSRSCl((\tilde{R}_2, E)^c) \neq 1_{(X,E)}$ implies that there exist a nsrsc set $(\tilde{S}, E)$ in $(X,\ \tau, E)$ such that $((\tilde{R}_2, E)^c) \subset (\tilde{S}, E) \subset 1_{(X,E)}$. Then $(\tilde{R}_1, E) \subseteq (\tilde{R}_2, E)^c \subset (\tilde{S}, E) \subset 1_{(X,E)} \Rightarrow (\tilde{R}_1, E) \subset (\tilde{S}, E) \subset 1_{(X,E)}$, which is a contradiction to $NSRSCl((\tilde{R}_1, E)) = 1_{(X,E)}$. Hence our assumption that $(X,\ \tau, E)$ is a $NSRS$-irresolvable space, is wrong. Hence $(X,\ \tau, E)$ is a $NSRS$-resolvable space. ■

Theorem 5.2. A nsts $(X,\ \tau, E)$ is a $NSRS$-resolvable space if $\bigcup_{i=1}^{n}(\tilde{R}_i, E) = 1_{(X,E)}$ where $NSRSInt((\tilde{R}_i, E)) = 0_{(X,E)}$.

Proof. $\bigcup_{i=1}^{n}(\tilde{R}_i, E) = 1_{(X,E)}$ where $NSRSInt((\tilde{R}_i, E)) = 0_{(X,E)}$, implies that $(\bigcup_{i=1}^{n}(\tilde{R}_i, E))^c = 0_{(X,E)}$. Then we have $\bigcap_{i=1}^{n}((\tilde{R}_i, E)^c) = 0_{(X,E)}$. Then there must be at least two non-zero disjoint Nss's $((\tilde{R}_i, E))^c$, $((\tilde{R}_j, E))^c$ in (X, τ, E). Hence $(((\tilde{R}_i, E))^c) + (((\tilde{R}_j, E))^c) \subseteq 1_{(X,E)}$. Therefore $(((\tilde{R}_i, E))^c) \subseteq (\tilde{R}_j, E)$ which implies that $NSRSCl(((\tilde{R}_i, E))^c) \subseteq NSRSCl((\tilde{R}_j, E))$. But $NSRSInt((\tilde{R}_i, E)) = 0_{(X,E)}$ implies that $NSRSCl(((\tilde{R}_i, E))^c) = 1_{(X,E)}$. Hence $1_{(X,E)} \subseteq NSRSCl((\tilde{R}_j, E))$ which implies that $NSRSCl((\tilde{R}_j, E)) = 1_{(X,E)}$. Also $NSRSInt((\tilde{R}_j, E)) = 0_{(X,E)}$ implies that $NSRSCl(((\tilde{R}_j, E))^c) = 1_{(X,E)}$. Therefore $(X,\ \tau, E)$ has a

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JCHR (2026) 16(1), 2717-2734 | ISSN:2251-6727

$NSRS$-dense set $(\tilde{R}_j, E)$ such that $NSRSCl(((\tilde{R}_j, E))^c) = 1_{(X,E)}$. Hence $(X,\ \tau, E)$ is a $NSRS$-resolvable space. ■

Theorem 5.3. If $(X,\ \tau, E)$ is $NSRS$-irresolvable if and only if $NSRSInt(R) \neq 0_{(X,E)}$ for all $NSRS$-dense sets $(\tilde{R}, E)$ in (X, τ, E).

Proof. Since $(X,\ \tau, E)$ is $NSRS$-irresolvable, for all $NSRS$-dense sets $(\tilde{R}, E)$ in $(X,\ \tau, E)$, we have $NSRSCl((\tilde{R}, E)^c) \neq 1_{(X,E)}$. Then $(NSRSInt((\tilde{R}, E)))^c \neq 1_{(X,E)}$ implies that $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$.

Conversely, let $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$ for each $NSRS$-dense set $(\tilde{R}, E)$ in $(X,\ \tau, E)$. Suppose that (X, τ, E) is $NSRS$-resolvable. Then there exists a $NSRS$-dense set $(\tilde{R}, E) \neq 0_{(X,E)}$ in $(X,\ \tau, E)$ such that $NSRSCl((\tilde{R}, E)^c) = 1_{(X,E)}$. Then we have $NSRSInt((\tilde{R}, E)) = 1_{(X,E)}$ and therefore $NSRSInt((\tilde{R}, E)) = 0_{(X,E)}$ which is a contradiction. Hence $(X,\ \tau, E)$ is a $NSRS$-irresolvable space. ■

6 Functions and neutrosophic soft regular semi irresolvable spaces

Definition 6.1. A function $f : (X,\ \tau, E) \to (Y,\ \sigma, E)$ is said to be weakly somewhat neutrosophic soft regular semi open function (for short, $WSNSRSO$)if for each $NSRS$-dense set $(\tilde{R}, E)$ in $(Y,\ \sigma, E)$ with $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$, we have that $f^{-1}((\tilde{R}, E))$ is also a $NSRS$-dense set in $(X,\ \tau, E)$.

The above definition leads to a characterization of $NSRS$-irresolvable space as follows:

Theorem 6.1. The following statements are equivalent for a nts $(Y,\ \sigma, E)$.

(1) $(Y,\ \sigma, E)$ is $NSRS$-irresolvable.

(2) For every nsts $(X,\ \tau, E)$, every $WSNSRSO$ function $f : (X,\ \tau, E) \to (Y,\ \sigma, E)$ is $SNSRS$-O.

Proof. (1) $\Rightarrow$ (2) Let $f : (X, \tau, E) \to (Y, \sigma, E)$ be a $WSNRSO$ function from a nts (X, τ, E) to a $NSRS$-irresolvable space $(Y,\ \sigma, E)$. Since $(Y,\ \sigma, E)$ is $NSSRS$-irresolvable space, $(Y,\ \sigma, E)$ has a pair of $NSRS$-dense sets $(\tilde{R}_1, E)$ and $(\tilde{R}_2, E)$ such that $(\tilde{R}_1, E) \nsubseteq ((\tilde{R}_2, E))^c$. Now $SNRSInt((\tilde{R}_1, E)) \neq 0_{(X,E)}$ and $NSRSInt((\tilde{R}_2, E)) \neq 0_{(X,E)}$.

For, if $NSRSInt((\tilde{R}_1, E)) = 0_{(X,E)}$ then, $(NSRSCl(((\tilde{R}_1, E))^c))^c = 0_{(X,E)}$. Now $(\tilde{R}_1, E) \supset ((\tilde{R}_2, E))^c \Rightarrow (\tilde{R}_2, E) \supset ((\tilde{R}_1, E))^c$.

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Therefore $NSRSCl((\tilde{R}_2, E)) \supset NSRSCl(((\tilde{R}_1, E))^c)$. In other words $(NSRSCl((\tilde{R}_2, E)))^c \subset (NSRSCl(((\tilde{R}_1, E))^c))^c = 0_{(X,E)}$. Then $1_{(X,E)} \subset NSRSCl((\tilde{R}_2, E))$ implies $1_{(X,E)} \subset 1_{(X,E)}$, which is a contradiction. Therefore $NSRSInt((\tilde{R}_1, E)) \neq 0_{(X,E)}$.

Similarly we can have $NSRSInt((\tilde{R}_2, E)) \neq 0_{(X,E)}$. Since f is $WSNSRSO$, $f^{-1}((\tilde{R}_1, E))$ and $f^{-1}((\tilde{R}_2, E))$ are $NSRS$-dense sets in (X, τ, E). Therefore by Theorem 4.1. f is $WSNSRSO$.

(2) $\Rightarrow$ (1) Suppose that nsts (Y, σ, E) is $NSRS$-resolvable. This means that there exists a pair of $NSRS$-dense sets $(\tilde{R}_1, E)$ and $(\tilde{R}_2, E)$ such that $(\tilde{R}_1, E) \subseteq [(\tilde{R}_2, E)]^c$. Define $f : (X, \tau, E) \to (Y, \sigma, E)$ to be the identity function. Then f is not $SNSRS$-O, since $f^{-1}((\tilde{R}_2, E))$ is not a $NSRS$-dense set in (X, τ, E). For, $f^{-1}((\tilde{R}_2, E)) = (\tilde{R}_2, E)$ and $(\tilde{R}_2, E) \subseteq ((\tilde{R}_1, E))^c \neq 1_{(X,E)}$. Then $(\tilde{R}_2, E) \subseteq ((\tilde{R}_1, E))^c \Rightarrow NSRSCl((\tilde{R}_2, E)) \subseteq NSRSCl(((\tilde{R}_1, E))^c)$. Since $((\tilde{R}_1, E))^c$ is nsrc set and hence nsrsc in (Y, σ, E), $NSRSCl((\tilde{R}_2, E)) \neq 1_{(X,E)}$. That is, $(\tilde{R}_2, E)$ is not a $NSRS$-dense set. We shall now show that f is $WSNSRSO$. Let $(\tilde{R}, E)$ be any $NSRS$-dense set in (Y, σ, E) such that $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$. Then $f^{-1}((\tilde{R}, E)) = (\tilde{R}, E)$. We have to show that $NSRSCl[f^{-1}((\tilde{R}, E))] = NSRSCl((\tilde{R}, E)) = 1_{(X,E)}$ in (X, τ, E). Now $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$ and $(\tilde{R}_1, E)$ is $NSRS$-dense implies that $(\tilde{R}, E) \nsubseteq ((\tilde{R}_1, E))^c$. Therefore $NSRSCl((\tilde{R}, E)) = 1_{(X,E)}$. That is, $(\tilde{R}, E)$ is $NSRS$-dense in (X, τ). This proves that f is $WSNSRSO$. Hence (2) $\Rightarrow$ (1) is proved. ■

Theorem 6.2. Let (X, τ, E) and (Y, σ, E) be any two nsts's. Let $f : X \to Y$ be a $SNSRS$-O function. If (X, τ, E) is a $NSRS$-irresolvable space, then (Y, σ, E) is a $NSRS$-irresolvable space.

Proof. Let $(\tilde{R}, E) \neq 0_{(X,E)}$ be an arbitrary neutrosophic set in Y such that $NSRSCl((\tilde{R}, E)) = 1_{(X,E)}$. We claim that $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$. Assume the contrary. That is, $NSRSInt((\tilde{R}, E)) = 0_{(X,E)}$. Then by Theorem 4.3., we have $NSRSInt(f^{-1}((\tilde{R}, E))) = 0_{(X,E)}$ in X. Now $(\tilde{R}, E)$ is $NSRS$-dense in Y, then by Theorem 4.1., we have $f^{-1}((\tilde{R}, E))$ is $NSRS$-dense in X.. Therefore for the $NSRS$-dense that $f^{-1}((\tilde{R}, E))$, we have $NSRSInt(f^{-1}((\tilde{R}, E))) = 0_{(X,E)}$ in X, which is a contradiction [since X is $NSRS$-irresolvable, by Theorem 5.3. $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$ for all $NSRS$-dense sets $(\tilde{R}, E)$ in X.] Hence we must have $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$ for all $NSRS$-dense sets $(\tilde{R}, E)$ in Y. Hence by Proposition 5.3., Y is a $NSRS$-irresolvable space. ■

Proposition 6.1. Let (X, τ, E) and (Y, σ, E) be any two nsts's and $f : X \to Y$ be a $SNSRSC$ and onto function. If Y is a $NSRS$-irresolvable space, then X is a $NSRS$-irresolvable space.

Journal of Chemical Health Risks
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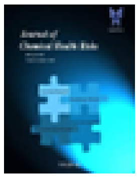
JCHR (2026) 16(1), 2717-2734 | ISSN:2251-6727



Proof. Let $(\tilde{R}, E) \neq 0_{(X,E)}$ be an arbitrary neutrosophic soft set in X such that $NSRSCl((\tilde{R}, E)) = 1_{(X,E)}$. We claim that $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$. Assume the contrary. That is, $NSRSInt((\tilde{R}, E)) = 0_{(X,E)}$. Then by Theorem 3.1., we have $NSRSInt(f((\tilde{R}, E))) = 0_{(X,E)}$. Now $(\tilde{R}, E)$ is $NSRS$-dense in X then by Theorem 3.1., we have $f((\tilde{R}, E))$ is $NSRS$-dense in Y. Therefore for the $NSRS$-dense set $f((\tilde{R}, E))$ in Y, we have $NSRSInt(f((\tilde{R}, E))) = 0_{(X,E)}$, which is a contradiction [since Y is $NSRS$-irresolvable, $NSRSInt((\tilde{S}, E)) \neq 0_{(X,E)}$ for all $NSRS$-dense sets $(\tilde{S}, E)$ in X.] Therefore we must have $NSRSInt((\tilde{R}, E)) \neq 0_{(X,E)}$ for all $NSRS$-dense sets $(\tilde{R}, E)$ in Y. Hence by Theorem 5.3., the nsts X is a $NSRS$-irresolvable space. ■

7 Application of Neutrosophic Soft Similarity Measure in Chemical Health Risk Assessment

Introduction

Chemical health risk assessment involves uncertainty in measurement data. Neutrosophic soft sets effectively model such imprecision using truth (μ), indeterminacy (σ) and falsity (ν) memberships.

Problem Description

Let $X = \{x_1, x_2, x_3\}$ denote three water sampling locations and $E = \{e_1, e_2, e_3\}$ represent lead, mercury and arsenic concentrations, respectively. Each neutrosophic soft value $\langle \mu, \sigma, \nu \rangle$ indicates the degrees of high risk, uncertain risk and low risk.

Neutrosophic Soft Decision Matrix

	e_1	e_2	e_3
x_1	$\langle 0.8, 0.1, 0.2 \rangle$	$\langle 0.6, 0.2, 0.3 \rangle$	$\langle 0.7, 0.2, 0.2 \rangle$
x_2	$\langle 0.5, 0.3, 0.4 \rangle$	$\langle 0.4, 0.3, 0.5 \rangle$	$\langle 0.3, 0.4, 0.6 \rangle$
x_3	$\langle 0.3, 0.4, 0.6 \rangle$	$\langle 0.2, 0.5, 0.7 \rangle$	$\langle 0.2, 0.6, 0.7 \rangle$

The ideal high-risk pattern is taken as $A_{\text{ideal}} = \langle 1, 0, 0 \rangle$.

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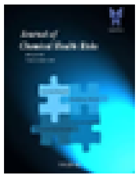

Similarity Results and Ranking

Based on the neutrosophic soft similarity computation, the similarity values obtained are:

$$S(x_1) = 0.767, \quad S(x_2) = 0.611, \quad S(x_3) = 0.422.$$

Thus, the ranking of the locations is

$$x_1 \succ x_2 \succ x_3.$$

Decision and Conclusion

Location x_1 exhibits the highest similarity to the ideal risk pattern and therefore represents the most critical contamination level requiring immediate remediation. Location x_2 shows moderate risk, while x_3 poses comparatively lower risk.

This application illustrates that the neutrosophic soft similarity measure effectively captures uncertainty and supports reliable environmental risk prioritisation.

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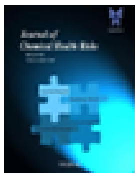

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